Forward dynamics of the double-wishbone suspension
mechanism using the embedded Lagrangian formulation

Vyankatesh Ashtekar
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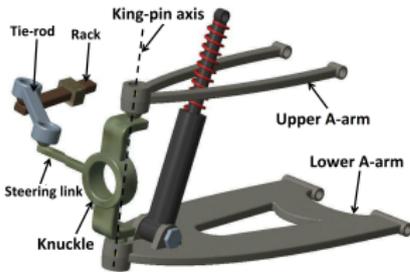
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The double wishbone (DWB) suspension

Source: mcdonaldbrosracing.com.au/
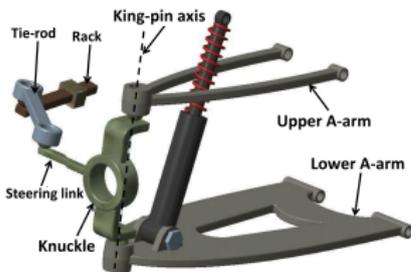

Source: [Reddy et al., 2016]

▶ Suspensions- what to analyse/design?
  ▶ Primary: vibration isolation
  ▶ Secondary: arrest bump steer, camber variation etc., during the suspension movement

▶ Features of the DWB suspension
  ▶ Used in high-end SUVs
  ▶ Independent suspension
  ▶ Short-Long Arm (SLA) suspension
  ▶ Parameter space large enough to achieve multiple design objectives

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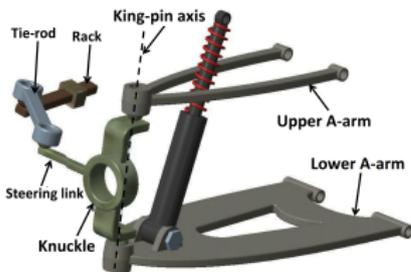

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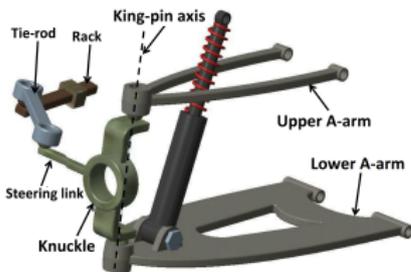Source: [Reddy et al., 2016]

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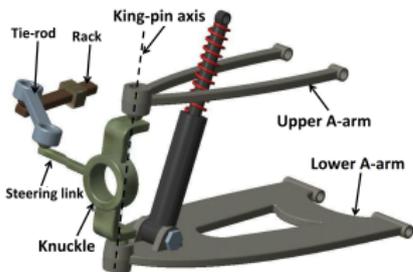Source: [Reddy et al., 2016]

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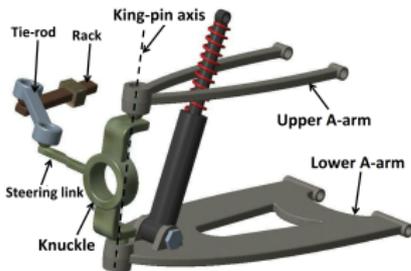Source: [Reddy et al., 2016]

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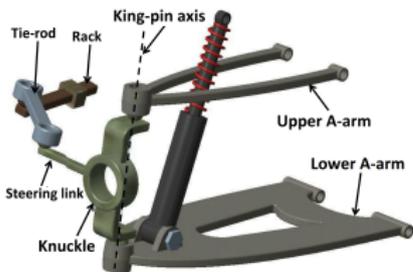Source: mcdonaldbrosracing.com.au/

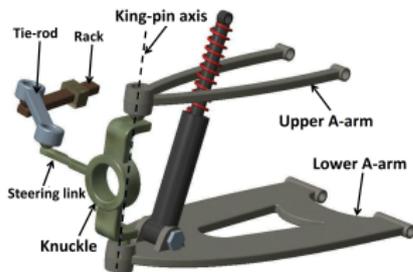Source: [Reddy et al., 2016]

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Geometry and kinematics

▶ **2-DoF mechanism**

▶ Independent variables: $\boldsymbol{\theta} = [y, s]^\top$

▶ Dependent variables: $\boldsymbol{\phi} = [\boldsymbol{c}^\top, \boldsymbol{\phi}_2^\top]^\top$
$\boldsymbol{c} = [c_1, c_2, c_3]^\top$
$\boldsymbol{\phi}_2 = [\theta, \psi, x, z, l_s, \delta]^\top$

▶ The forward kinematics (FK) problem: find all possible $\boldsymbol{c}$ given $\boldsymbol{\theta}$

▶ Configuration space:
$\boldsymbol{q} = [\boldsymbol{\theta}^\top, \boldsymbol{\phi}^\top]^\top \in \mathbb{R}^{11}$

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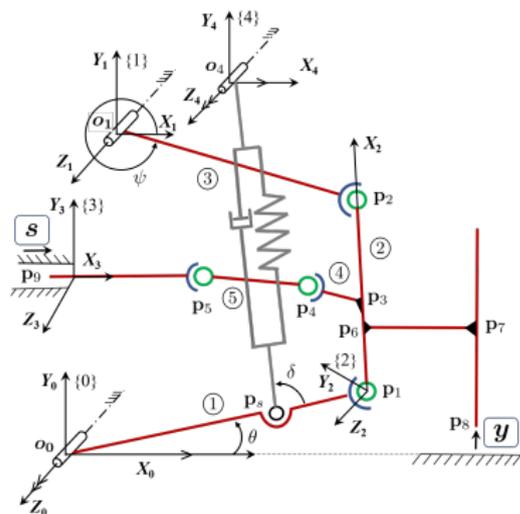- ▶ The forward kinematics (FK) problem: find all possible $\boldsymbol{c}$ given $\boldsymbol{\theta}$

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Forward dynamics of the DWB mechanism
Lagrangian methods

1. **Configuration space**
   The equations of motion involve matrices of size 11×11

2. **Augmented form**
   DAEs of index 2, matrices of size 11×11, stability checks necessary

3. **Embedded form**
   ▶ Equivalent unconstrained description of the original system
   ▶ Only the 2 *active variables* are integrated
   ▶ Remaining 9 *passive variables* updated via FK
   ▶ 64 solutions to the FK problem. Computationally expensive

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Scope of this work

▶ Dynamic model derived from the most general geometry of the mechanism

▶ Reduction in the size of matrices: 11×11 to 2×2, using the embedded Lagrangian form

▶ Numerical integration: the Runge-Kutta method of order 4

▶ Root-tracking algorithm developed to expedite the FK computations

▶ Implementation in CAS Mathematica

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Forward kinematics
Loop closure constraints (LCC)

Vyankatesh Ashtekar and Sandipan Bandyopadhyay, Department of Engineering Design, IIT Madras.

Forward dynamics of the double-wishbone suspension mechanism... [Asian MMS '18, 17-19 Dec., 2018]

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Forward kinematics

Loop closure constraints (LCC)

▶ $\boldsymbol{o}_0 \boldsymbol{p}_1 \boldsymbol{p}_2 \boldsymbol{o}_1 \boldsymbol{o}_0$: The spatial four bar loop

▶ LCC: $\boldsymbol{\eta}_1 = [\eta_{1x}, \eta_{1y}, \eta_{1z}]^\top = \boldsymbol{0}$

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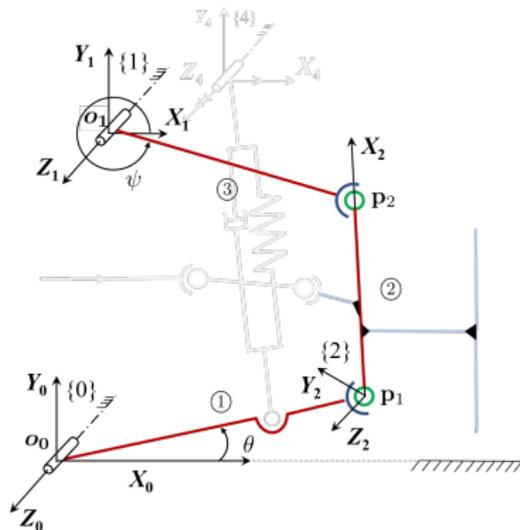
Forward kinematics

Loop closure constraints (LCC)

▶ $\boldsymbol{o}_0\boldsymbol{p}_1\boldsymbol{p}_4\boldsymbol{p}_5\boldsymbol{p}_9\boldsymbol{o}_0$: The spatial five bar loop

▶ LCC: $\eta_2 = (\boldsymbol{p}_5 - \boldsymbol{p}_4) \cdot (\boldsymbol{p}_5 - \boldsymbol{p}_4) - l_5^2 = 0$

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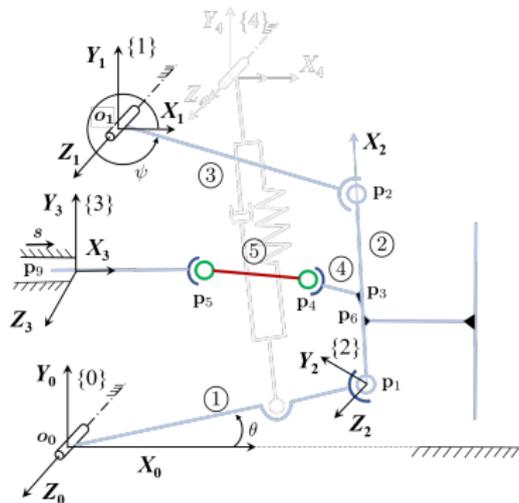
Forward kinematics

Loop closure constraints (LCC)

- ▶ Connecting the road input to the mechanism
- ▶ LCC: $\boldsymbol{\eta}_3 = [\eta_{3x}, \eta_{3y}, \eta_{3z}]^\top = \mathbf{0}$

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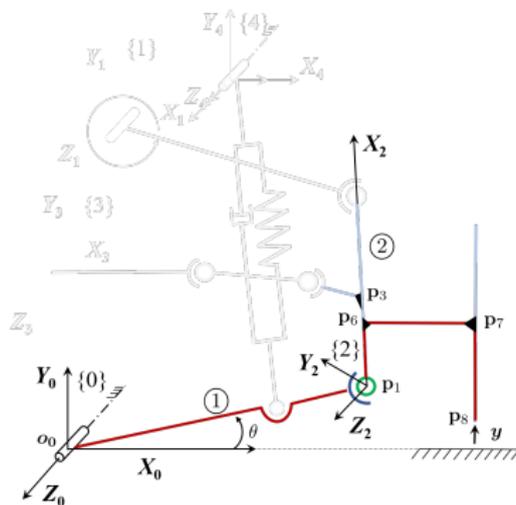
Forward kinematics

Loop closure constraints (LCC)

- ▶ Connecting the spring and the damper to the lower A-arm
- ▶ LCC: $\boldsymbol{\eta}_4 = [\eta_{4x}, \eta_{4y}]^\top = \mathbf{0}$

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Forward kinematics (contd...)

- $\boldsymbol{\eta}(\boldsymbol{q}) = [\boldsymbol{\eta}_1^\top, \eta_2, \boldsymbol{\eta}_3^\top, \boldsymbol{\eta}_4^\top]^\top$

$$\boxed{\boldsymbol{\eta}(\boldsymbol{q})} \xrightarrow{\times \boldsymbol{\phi}_2} \boxed{\boldsymbol{f}(\boldsymbol{c})}$$

- $\boldsymbol{f}(c_1, c_2, c_3) = [f_1, f_2, f_3]^\top$:
  size of quartic equations in closed form: $\sim 1.3\,\mathrm{MB}$
- 64 finite solutions, ≤ 4 real solutions[1]
- Solution to $\boldsymbol{\phi}_2$: unique, closed form expressions

[Reddy et al., 2016]

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[1]This is an observation only, proof is not available.

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Dynamics

The embedded Lagrangian form

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau^{\mathrm{nc}} + \tau^{\mathrm{c}}$$

(non-singular cases)

$$M_\theta(q)\ddot{\theta} + C_\theta(q,\dot{q})\dot{\theta} + G_\theta(q) = \tau_\theta$$

▶ $M \in \mathbb{R}^{11 \times 11},\ M_\theta \in \mathbb{R}^{2 \times 2}$

▶ Expressions in closed form derived for M, C and G

▶ $\tau^{\mathrm{nc}} = J_{v_{p_s}}^\top F_d, \quad F_d = -C_d \dot{l_s}$

▶ $M_\theta = J_{q\theta}^\top M J_{q\theta},\ C_\theta = J_{q\theta}^\top (M \dot{J_{q\theta}} + C J_{q\theta}),$

$G_\theta = J_{q\theta}^\top G$ and $\tau_\theta = J_{q\theta}^\top \tau^{\mathrm{nc}}$

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$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) = \boldsymbol{\tau}^{\mathrm{nc}} + \boldsymbol{\tau}^{\mathrm{c}}$$

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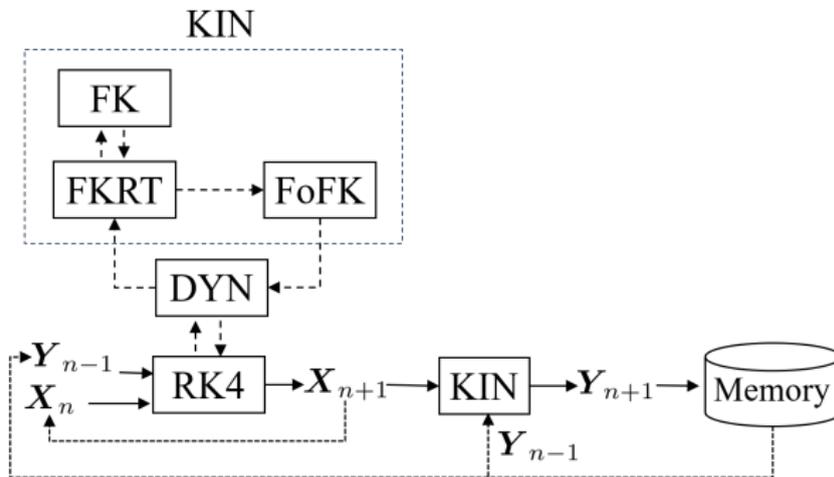
The embedded Lagrangian form

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau^{\mathrm{nc}} + \tau^{\mathrm{c}}$$

↓ (non-singular cases)

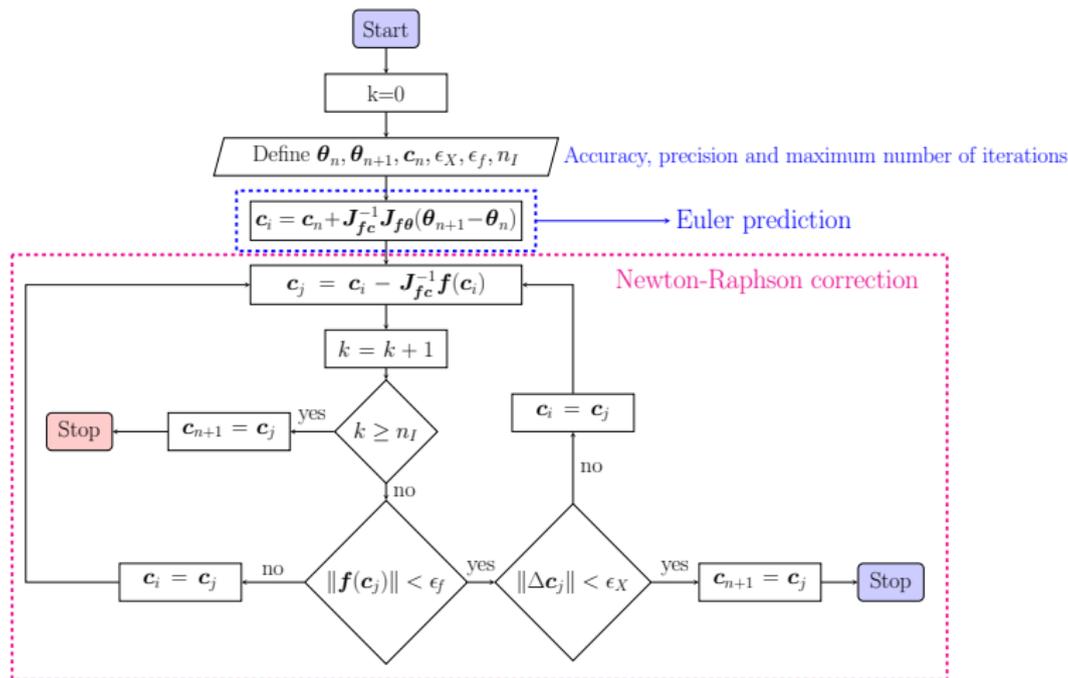$$M_{\theta}(q)\ddot{\theta} + C_{\theta}(q,\dot{q})\dot{\theta} + G_{\theta}(q) = \tau_{\theta}$$

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- Expressions in closed form derived for M, C and G

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- $M_{\theta} = J_{q\theta}^{\top} M J_{q\theta}$, $C_{\theta} = J_{q\theta}^{\top}(M\dot{J_{q\theta}} + C J_{q\theta})$,

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Dynamics

The embedded Lagrangian form

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau^{\text{nc}} + \tau^{\text{c}}$$

$\downarrow$ (non-singular cases)

$$M_\theta(q)\ddot{\theta} + C_\theta(q,\dot{q})\dot{\theta} + G_\theta(q) = \tau_\theta$$

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The algorithm

▶ $\dot{\boldsymbol{X}} = \begin{pmatrix} \dot{\boldsymbol{\theta}} \\ \ddot{\boldsymbol{\theta}} \end{pmatrix} = \begin{pmatrix} \dot{\boldsymbol{\theta}} \\ \boldsymbol{M_\theta}^{-1}(\boldsymbol{\tau_\theta} - \boldsymbol{C_\theta}\dot{\boldsymbol{\theta}} - \boldsymbol{G_\theta}) \end{pmatrix} = \boldsymbol{f}_d(\boldsymbol{X})$

▶ $\boldsymbol{Y} = [\boldsymbol{q}^\top, \dot{\boldsymbol{q}}^\top]^\top$

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Numerical integration

Faster FK computations using root-tracking

Simulation details

Source: superchevy.com

Source: ssvc.org.uk

▶ Initial configuration: start of jounce

▶ Exponential damping: $\boldsymbol{\tau}^{\mathrm{nc}}(t) = -F_d \, \dot{l}_s$

▶ Spring force considered in potential energy term $\boldsymbol{G}$

▶ Simulation time: $1.5\,\mathrm{s}$

▶ Time step: $0.002\,\mathrm{s}$

▶ Time taken for 750 steps: $324\,\mathrm{s}$ on Intel® Core™ i7-7500U CPU @ $3.46\,\mathrm{GHz}$

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▶ Simulation time: $1.5\,\mathrm{s}$

▶ Time step: $0.002\,\mathrm{s}$

▶ Time taken for 750 steps: $324\,\mathrm{s}$ on
Intel® Core™ i7-7500U CPU @ $3.46\,\mathrm{GHz}$

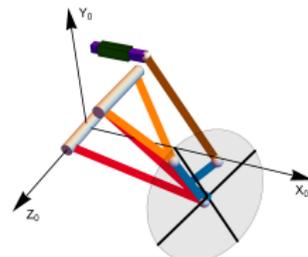| Overview	Dynamics	Numerical integration	Illustration	Summary
○○○○	○○○	○○	●○○○○	○○○

Illustration

Simulation details

Source: superchevy.com

Source: ssvc.org.uk

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Vyankatesh Ashtekar and Sandipan Bandyopadhyay, Department of Engineering Design, IIT Madras.

Forward dynamics of the double-wishbone suspension mechanism... [Asian MMS '18, 17-19 Dec., 2018]
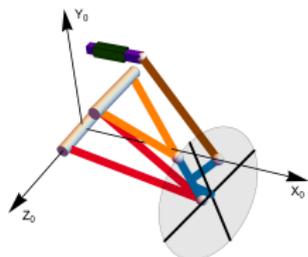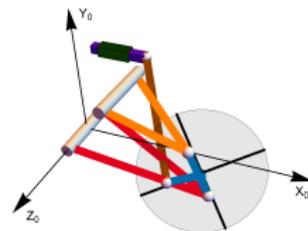
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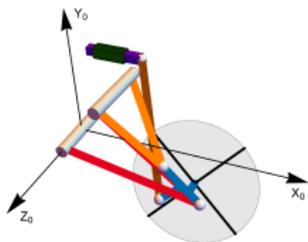▶ Time taken for 750 steps: $324\,\mathrm{s}$ on
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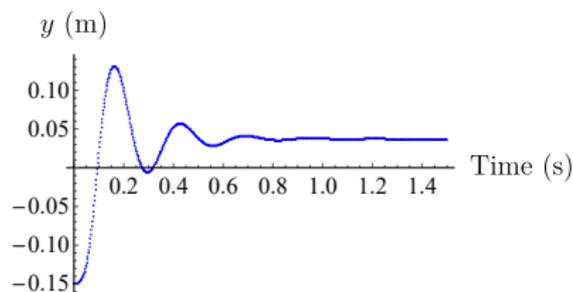▶ Time taken for 750 steps: $324\,\mathrm{s}$ on
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Overview
0000

Dynamics
000

Numerical integration
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Illustration
0●000

Summary
000

Illustration

Initial configurations for $y(0) = -150$, and $s(0) = 0$

Overview	Dynamics	Numerical integration	Illustration	Summary
oooo	ooo	oo	○●○○○	ooo

Illustration

Initial configurations for $y(0) = -150$, and $s(0) = 0$

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Overview
○○○○

Dynamics
○○○

Numerical integration
○○

Illustration
○○●○○

Summary
○○○

Illustration

Free response of the suspension for
$y(0) = -150, s(0) = 0, \dot{y}(0) = 0$ and $\dot{s}(0) = 0$

Overview
○○○○

Dynamics
○○○

Numerical integration
○○

Illustration
○○○●○

Summary
○○○

Illustration

Validation of the results

$e_0(\times 10^{-14})$ (m)

3.0
2.5
2.0
1.5
1.0
0.5

Time (s)

0.2 0.4 0.6 0.8 1.0 1.2 1.4
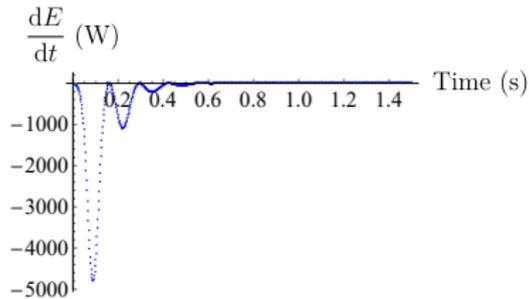
▶ Degree of satisfaction of the LCC: $e_0 = \|\boldsymbol{\eta}(\boldsymbol{q})\|$

▶ Net power flux: $e_1 = \frac{\mathrm{d}E}{\mathrm{d}t} - \boldsymbol{Q}^{\mathrm{nc}} \cdot \dot{\boldsymbol{q}}$
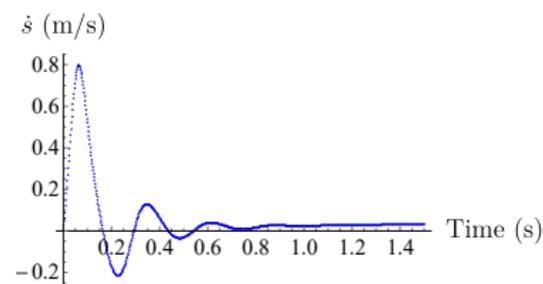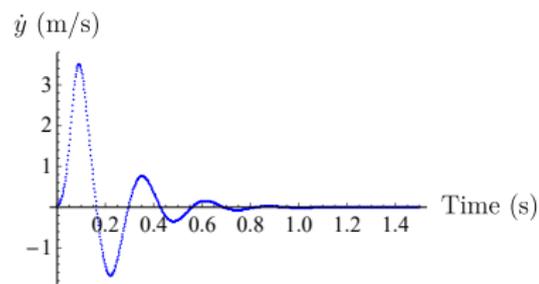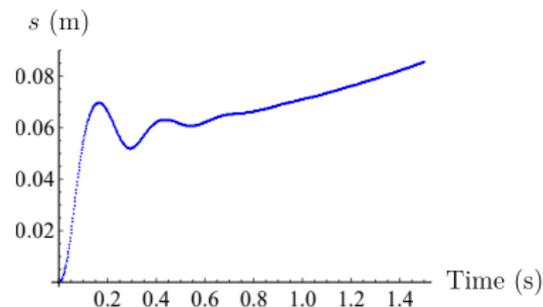
Validation of the results

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- Net power flux: $e_1 = \frac{\mathrm{d}E}{\mathrm{d}t} - \boldsymbol{Q}^{\mathrm{nc}} \cdot \dot{\boldsymbol{q}}$

Overview
oooo

Dynamics
ooo

Numerical integration
oo

Illustration
oooo●

Summary
ooo

Illustration

Validation of the results (contd...)

Summary

▶ Eleven ODEs reduced to two ODEs

▶ The kinematic constraints are solved in algebraic manner as opposed to via larger system of ODEs

▶ Kinematic constraints satisfied
  ▶ to a higher accuracy
  ▶ independent of the length of the simulation– no drift

▶ Proposed root-tracking method guarantees convergence, notifies in case of singularities

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Future extensions

▶ Algorithms would be implemented in C/C++ to bring forth the computational advantages

▶ The dynamics module to be used in the suspension design optimisation studies

Overview
○○○○

Dynamics
○○○

Numerical integration
○○

Illustration
○○○○○

Summary
○○●

Summary

Thank you for your attention!

Questions/comments?

Vyankatesh Ashtekar and Sandipan Bandyopadhyay, Department of Engineering Design, IIT Madras.

Forward dynamics of the double-wishbone suspension mechanism... [Asian MMS '18, 17-19 Dec., 2018]

📄 Attia, H. A. (2002).
Dynamic modelling of the double wishbone motor-vehicle
suspension system.
European Journal of Mechanics - A/Solids, 21(1):167–174.

📄 Reddy, K. V., Kodati, M., Chatra, K., and Bandyopadhyay,
S. (2016).
A comprehensive kinematic analysis of the double wishbone
and MacPherson strut suspension systems.
Mechanism and Machine Theory, 105:441–470.

📄 Uchida, T. and McPhee, J. (2012).
Driving simulator with double-wishbone suspension using
efficient block-triangularized kinematic equations.
*Multibody System Dynamics*, 28(4):331–347.

Assumptions

► All the links are rigid (including tyre).

► All the joints are ideal, i.e., frictionless and rigid.

► Universal joints at the end of tie-rods are assumed as spherical joints. This results in an idle degree-of-freedom in the tie-rod. Energy corresponding to the idle degree-of-freedom is ignored in the derivation of the equations of motion.

State of the art

- ▶ [Attia, 2002] have transformed from task space variables to joint space.
- ▶ [Uchida and McPhee, 2012] have implemented the embedded Lagrangian formulation. However, a surrogate input is used instead of actual road input.

Forward kinematics (contd...)

▶ FK1: Analytical solution to $\boldsymbol{f}(c_1, c_2, c_3) = \boldsymbol{0}$. Sixty four finite solutions. Not more than four found to be real

▶ FK2: Compute $\boldsymbol{\phi}_2$ for given $\boldsymbol{c}$

▶ First integration step: FK block is executed. Next step onwards FKRT block is used

Derivation of the equations of motion
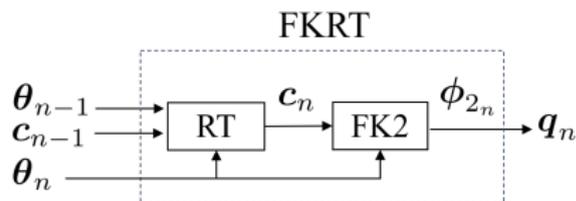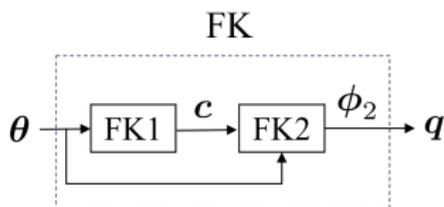
Parameter	Symbol	Value
Equivalent length of lower A-arm	l_1	420
Length of the KPA	l_2	163
Equivalent length of upper A-arm	l_3	260
Length of tie-rod	l_5	520
Euler angles relating {1} to {0}	$(\alpha_1, \alpha_2, \alpha_3)$	(-2,-1,3)
Euler angles relating {3} to {0}	$(\beta_1, \beta_2, \beta_3)$	(2,1,-3)
Link ④ vector w.r.t. {2}	(l_{4x}, l_{4y}, l_{4z})	(0,103,0)
Position of the origin of {0}	$\boldsymbol{o}_0$	(0,0,0)
Position of the origin of {1}	$\boldsymbol{o}_1$	(130,160,,0)
Initial position of $\boldsymbol{p}_5$	(p_{5x}, p_{5y}, p_{5z})	(10,20,-400)
Initial position of $\boldsymbol{p}_7$	(p_{7x}, p_{7y}, p_{7z})	(480,50,0)
Initial position of $\boldsymbol{p}_8$	(p_{8x}, p_{8y}, p_{8z})	(480,-150,-4.801)
$^2\boldsymbol{l}_6$ vector w.r.t. {2}	(l_{6x}, l_{6y}, l_{6z})	(-209,7,-30)
$\|\boldsymbol{p}_1 - \boldsymbol{p}_3\|$ as fraction of $\boldsymbol{l}_2$	r	0.473
$\|\boldsymbol{p}_1 - \boldsymbol{p}_6\|$ as fraction of $\boldsymbol{l}_2$	ρ	0.312
$\|\boldsymbol{p}_1 - \boldsymbol{p}_s\|$ as fraction of $\boldsymbol{l}_2$	σ	0.642

Source:[Attia, 2002]

Parameter	Symbol	Value
Mass of lower A-arm	m_1	1
Mass of link of upper A-arm	m_3	1
Mass of knuckle and steering link	m_2	8
Mass of wheel	m_w	22
Mass of tie-rod	m_5	1
Mass of steering rack	m_6	6.5/2
Mass of spring and damper combined	m_{sd}	4
Inertia of lower A-arm	(I_{1x}, I_{1y}, I_{1z})	(0.028,0.002,0.03)
Inertia of upper A-arm	(I_{3x}, I_{3y}, I_{3z})	(0.028,0.002,0.03)
Inertia of knuckle and steering link	(I_{2x}, I_{2y}, I_{2z})	(1.6,1.6,1.6)
Inertia of wheel	(I_{wx}, I_{wy}, I_{wz})	(2,1.35,2)
Inertia of tie-rod (about one of the ends)	(I_{5x}, I_{5y}, I_{5z})	$(0, \frac{m_5 l_5^2}{3}, \frac{m_5 l_5^2}{3})$
Coordinates of c.o.m. of link ①	$(rc_{1x}, rc_{1y}, rc_{1z})$	(0.14,0,0)
Coordinates of c.o.m. of link ②	$(rc_{2x}, rc_{2y}, rc_{2z})$	(0.0815,0.0035,-0.01)
Coordinates of c.o.m. of link ③	$(rc_{3x}, rc_{3y}, rc_{3z})$	(0.13,0,0)
Coordinates of c.o.m. of link ⑤	$(rc_{5x}, rc_{5y}, rc_{5z})$	$(l_5/2,0,0)$
Coordinates of c.o.m. of the wheel	(rcw_x, rcw_y, rcw_z)	(0.0815,-0.0035,0)
Spring constant (N/m)	K	5.11×10^4
Coefficient of damping (N-s/m)	C_d	0.1×10^4
Free length of the spring	l_s	482

Source: [Attia, 2002]

Source: https://www.ssvc.org.uk/phpbb/viewtopic.php?t=47345